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Modeling the reliability of repairable systems in the aviation industry

G.R. Weckman[a,*], R.L. Shell[b], J.H. Marvel[c]

[a]Department of Mechanical & Industrial Engineering, Campus Box 191, Texas A&M University — Kingsville, Kingsville, TX 78363, USA

[b]Department of Mechanical, Industrial & Nuclear Engineering, University of Cincinnati, Cincinnati, OH 45221-0072, USA

[c]Padnos School of Engineering, Grand Valley State University, Grand Rapids, MI 49504, USA

Abstract

The jet engine is an example of a complex system that periodically requires repair or restoration. This paper discusses how the Weibull process, a non-homogenous Poisson process, can be used as a new approach in modeling jet engine life. The Weibull process can be a very useful tool in modeling repairable systems. The removal characteristics are estimated by collecting actual field data based on the engine age and operating environment. The process parameters are estimated using methodology that is based on data generated from multiple systems. This analysis includes an example of a jet engine application illustrating how the model predictions compare to actual events. The overall capability of the model is measured by examining both data fit and forecasting accuracy. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Reliability modeling; Weibull process; Repairable systems; Non-homogenous Poisson process; Maintenance planning

1. Introduction

Since the days of deregulation, the aviation industry has considered many methods, techniques, and procedures to reduce operational costs. A significant portion of these operational costs is the resource allocated to jet engine repairs. In order to effectively allocate these resources, the industry needs to be able to accurately forecast engine removals. The jet engine has many failure modes that can cause the removal characteristics to have a substantial amount of variation in the expected time-on-wing (TOW) causing inaccuracies in the forecast. This paper focuses on the development of a forecasting methodology based on actual field data utilizing the Weibull process.

As noted by Ascher and Feingold (1984), Crow (1990), Love and Guo (1993) and Rigdon and Basu

* Corresponding author.

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(1989), very little literature has been published on the reliability of repairable systems. Typically, the reliability literature has been focused on non-repairable reliability utilizing the "renewal theory". The ordinary renewal process, being the simplest possible, is represented by a system composed of only one component. The process begins by placing one component into operation, then, when it fails, it is replaced by another new component and the system is repaired to a "same-as-new" condition (Ascher & Feingold, 1984). This type of process does not change as the system ages and is modeled by a homogenous Poisson process (HPP). The HPP system is neither improving nor wearing out with age and has a constant failure rate.

However, in complex machinery such as a jet engine, the systems are generally not replaced but are repaired when they fail. In this case, the usual non-repairable methodologies are simply not appropriate for repairable systems and the renewal process should not be used since the required refurbishment will typically not achieve a "same-as-new" status (Crow, 1993). The general procedure used to estimate this complicated failure process is by modeling through the use of a simpler failure process that would still provide practical results. The model typically used for this type of repairable system analysis is a non-homogenous Poisson process (NHPP) (Crow, 1993; Rigdon & Basu, 1989). Modeling system deterioration or system growth requires the use of an NHPP system that could improve or deteriorate with system age. In fact, in the reliability analysis of repairable systems the major interest is in the probability of system failure as a function of system age. The NHPP has a very important characteristic in which the intensity function $u(t)$ depends on cumulative system operating time, "global time", and not necessarily on the previous time of the most recent failure "local time" (Ascher & Feingold, 1984).

The Weibull process has been referred to by many different terms such as the Power Law process, Weibull restoration process, NHPP with Weibull intensity function, Weibull Poisson process and more recently as the Power Law NHPP. In this paper the term Weibull process is used based on its more frequent use in the existing literature.

2. Jet engine characteristics

The jet engine represents the leading edge of technology, advanced manufacturing, quality control, design evaluation and extensive testing. This machinery, with its new hardware and systems can achieve very high standards of reliability. The uncertainty of an engine failure or removal is dependent on a number of external and internal factors (Moss, 1991):

- component-specific factors (design, manufacturing);
- operational factors (pressure, temperature);
- environmental factors (ambient conditions, temperature, humidity);
- maintenance factors (servicing frequency, overhaul strategy).

After the engines have flown, typically 3–4 years before removal from the aircraft, the hardware will undergo wear, fretting, fatigue, erosion, corrosion, distortion and other forms of distress. Some of these deterioration factors, if not refurbished properly, can limit the ability of the engine to stay on-wing during its second and subsequent installations (Kleinert & Gregg, 1990; Lewis, 1987). Upon replacement, the engine is sent to a repair/overhaul facility. When the cause for removal requires penetration

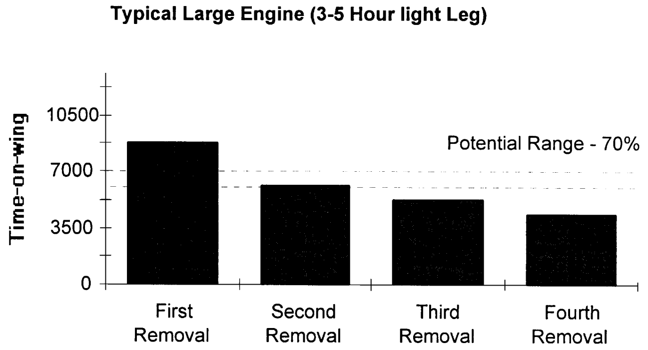

Fig. 1. TOW deterioration (*Source:* Kleinert, 1990).

into a module (standardized breakdown of the engine into workable sections) then it is classified as a shop visit (SV).

It is at this point that different maintenance philosophies and initial engine designs produce various levels of restored reliability. With good workscope planning and shop methods, an engine can be restored and achieve a subsequent TOW of approximately 70% of first run (new) capability. However, in many cases the TOW will deteriorate to as low as 50% of first run capability as the engine ages (Kleinert, 1990) (see Fig. 1).

The types of performance deterioration for turbine jet engines are (Diakunchak, 1992):

- recoverable with cleaning/washing (accumulation dirt, dust, pollen, particles in gas path, etc.);
- non-recoverable with cleaning/washing (deposits remain after cleaning/wash, flow path damage, erosion, corrosion, etc.);
- permanent, not recoverable after refurbishment (as closely to "as new" but loss due to eccentricity in clearances, increase leakage paths, surface roughness, distortion in platforms, etc.).

These factors account for the inability to restore the reliability of a jet engine as it ages resulting in a condition *not* "same-as-new". Love and Guo (1993) define this condition as imperfect repair.

Another characteristic that emerges is the potential for reliability growth in a newly developed engine program as proposed by Duane (1964). This concept was originally based on the "learning curve" theory developed by Wright (1936). In practically every new engine development program, the reliability tends to improve during design, development, testing and actual use. This is due to the continuing engineering effort to improve the design, including manufacture and operation of the hardware. This process is typified by the development cycle based on fixing or modifying the design as new information is gained and knowledge from prior testing (test–fix–test–fix process) is assessed. These growth models are very useful in predicting the future reliability or failure rates (Lewis, 1987; Nelson, 1990; O'Connor, 1991).

3. Weibull process as a model for repairable systems

In terms of a repairable system, the constant intensity (another term for instantaneous failure rate), represented by $\lambda \Delta t$ where λ is the constant failure rate, implies that there is no improvement or wearout

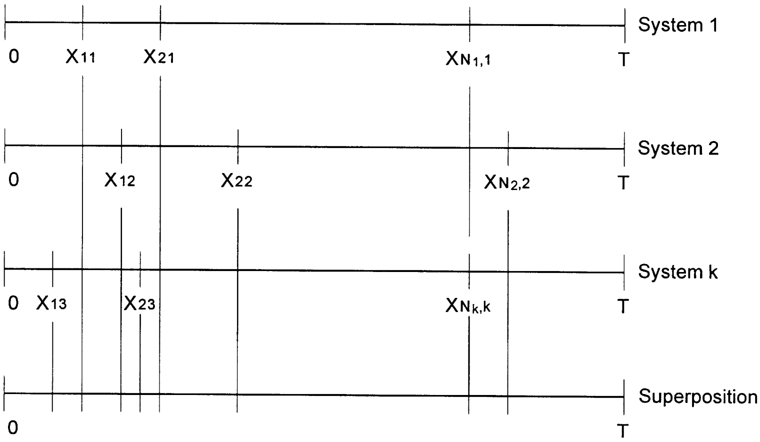

Fig. 2. Superposition of K systems (*Source:* Crow, 1993).

with age, characterized as the HPP. The NHPP is a generalization of the HPP that allows for a change in the intensity as a function of system age t. In this case, the $\lambda \Delta t$ is replaced by $u(t)\Delta t$ which is the approximate probability that a failure will occur between age t and $t + \Delta t$ of the system (Crow, 1993).

The Weibull process is the NHPP with an intensity function of the form:

$$u(t) = \lambda \beta t^{\beta-1} \qquad \text{for } t > 0 \tag{1}$$

where $\lambda > 0$, $\beta > 0$, t is the system's age and $u(t)$ is of the same form as the failure rate for a Weibull distribution. However, as noted by Crow, the Weibull distribution terminology, estimation and other statistical procedures do not apply to this NHPP process. When $\beta > 1$ the intensity function is increasing (representing system deterioration) and $\beta < 1$ implies a decreasing function (representing system improvement). The NHPP reverts to an HPP when $\beta = 1$ when the intensity function is equal to a constant λ.

As Crow pointed out, when analyzing complex repairable systems, the failure data are collected from the field and is generated by multiple copies of the system operated over different time periods. One approach in modeling is to take the number of different systems k, each with an operating time of at least T and pool the behavior. The time period, in this case, would be represented as $(0,T)$. It must be assumed that each system is treated as a copy and be representative of the same population. This method can then be adapted for multiple system behavior by superpositioning the failure times for the k copies onto a single time line (Crow, 1993) (see Fig. 2).

This allows, for the superposition system, in which a failure is noted each time any one of the k systems fails, the intensity function $u^*(t)$ of the system is represented by (Crow, 1993):

$$u^*(t) = ku(t) = k\lambda\beta t^{\beta-1} \qquad \textit{for } 0 < t < T. \tag{2}$$

In this case, the $u^*(t)$ would have a scale parameter $\lambda^* = k\lambda$ but with the same shape parameter of β. The same procedures used for estimating parameters for a single system can be applied to the superposition intensity function $u^*(t)$ (Crow, 1993).

4. Estimation of the intensity function — time truncated data

For the superposition system where data are collected over a specified time interval, the analysis is called time truncated. For a time truncated analysis, the maximum likelihood (ML) estimates of the intensity function is given by (Crow, 1993):

$$\hat{u}^*(t) = \hat{\lambda}^* \hat{\beta} t^{\hat{\beta}-1} \qquad \text{for } t > 0. \tag{3}$$

In the case where there are k systems with N total failures, N is calculated as:

$$N = \sum_{q=1}^{k} N_q \tag{4}$$

and for each of the k systems, the failures are denoted as X_{ik} where i represented the number of failures in a given system, then it follows that the MLE can be represented for the parameters (λ^* and β) as:

$$\hat{\lambda}^* = \frac{N}{T^{\hat{\beta}}} \tag{5}$$

and

$$\hat{\beta} = \frac{N}{\sum_{q=1}^{k} \sum_{i=1}^{N_q} \ln\left[\frac{T}{X_{iq}}\right]} \tag{6}$$

where the MLE for the intensity functions for each system could be represented by:

$$\hat{u}(t) = \hat{\lambda} \hat{\beta} t^{\hat{\beta}-1} = \frac{1}{k} \hat{\mu}^*(t) \tag{7}$$

and as previously noted, $\lambda^* = k\lambda$, then:

$$\hat{\lambda} = \frac{N}{kT^{\hat{\beta}}}. \tag{8}$$

5. A jet engine application

The engine removal data was obtained from two different airline databases over a period of years with a substantial sample size of removals. (Note: due to proprietary reasons the data have been masked.) They are referred to as Airline "A" and Airline "B" to maintain the confidentiality of the operators involved. In choosing these two airlines, different operating environments were selected. The database includes key information such as time since new (TSN), time since last shop visit (TSSV or TOW), engine delivery and removal dates (Event). All of the historical data were used to develop the model, while a more recent update for the following 27 months of the airline's operation was used to evaluate the effectiveness of this forecasting methodology. All the analysis regarding this application utilized Microsoft Excel® for parameter estimation and ProModel® for simulation design.

The model utilized the Weibull process methodology discussed previously and the analysis was

Table 1
Sample data for $k = 25$ jet engines operating for $T = 550$ flying hours

Engine #	Cuml. failure time				Total failures
1	150	407	526		3
2	291	439			2
3	93	179	357	547	4
4	53	203	275	395	4
5	2	188	265	364	4
6	65	250	370	550	4
7	183	290	545		3
8	144	338	523		3
9	223	531			2
10	197	367			2
11	187	215	357		3
12	197	356			2
13	213	370			2
14	171	332	539		3
15	197	312	435		3
16	200	312			2
17	262	509			2
18	255	395			2
19	286	452			2
20	206	383	479		3
21	179	444			2
22	232	488			2
23	165	417			2
24	155	373			2
25	203	292	469		3

divided into the following key steps:

- Year of delivery separation
- Parameter estimation
- Seasonal effect
- Simulation design

5.1. Year of delivery separation

The NHPP process was constructed to represent the engine removal process of sequential replacement by use of a Weibull power function. First, the removals were separated into years of delivery to account for design improvements as depicted previously by the Duane growth models. Each of these subpopulations was analyzed separately and the start times were indexed to begin at the same time ($t = 0$). The initial or "first year of delivery" intensity function was constructed analyzing only removals occurring during a specified window of time ($T = 550$). In this case, the time truncated MLEs were used to estimate the parameters. Table 1 shows the field data collected.

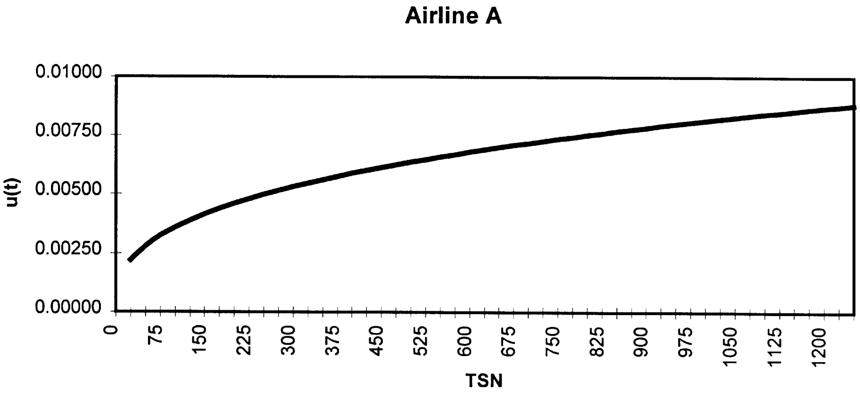

Fig. 3. Intensity function — first year of delivery.

5.2. Parameter estimation

Considering Eqs. (5) and (6), the estimated value for the parameters in the superposition intensity function was as follows:

$$\hat{\lambda}^* = 0.0005167 \qquad \text{and} \qquad \hat{\beta} = 1.355$$

where

$$\hat{u}(t) = 0.0005167 \times 1.355 \times t^{0.355}$$

represents the intensity function assigned to all jet engines delivered during the first year of operation (see Fig. 3).

The above intensity function can also be displayed as the probability of completing a mission (TSSV) based on the age (TSN) of the engine (see Fig. 4).

This same methodology was used to determine subsequent "years of delivery". Unfortunately, after

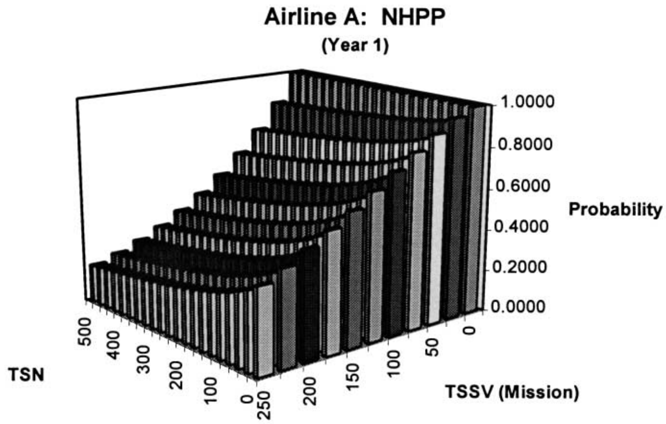

Fig. 4. Mission probability (TSSV) with respect to engine age (TSN).

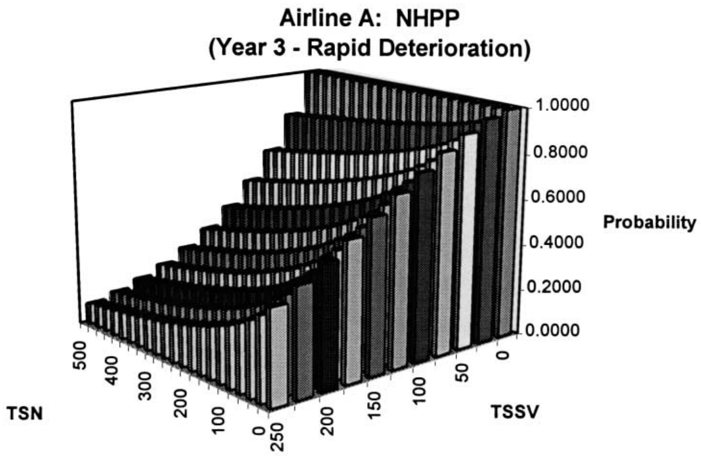

Fig. 5. Rapid deterioration with respect to TSSV and TSN.

the first 3 years of deliveries, the amount of data available for analysis was insufficient to determine the values of the functional parameters. In the case of latter years (three or more), only a few engines had three or more shop visits, which did not accurately model sequential replacements (counting). This is an unfortunate problem associated with new engine programs. To obtain sufficient number of shop visits would take another decade of data collection. The resulting intensity function describing this situation deteriorated much too rapidly (see Fig. 5).

The above function indicates that an engine would deteriorate to less than a 10% mission probability by 125 time-since-shop-visit (TSSV) and 250 time-since-new (TSN) that was not at all reflective of actual results.

Since data were not available beyond the first two years of delivery the following steps were taken in order to construct a reasonable intensity function. The third year, due to its similarity to the second, was assigned the same parametric values. For years 4 and higher, a series of estimated TSSVs were determined based on knowledge of the engine and the expected mature values. The calculations used to establish the projected TSSV curve are shown in Table 2 and the resulting intensity function is displayed in Fig. 6.

5.3. Simulation design

The basic simulation design assumes that the Weibull process could be used to determine the TSSV

Table 2
Estimated mature engine performance

	SV#							
	1	2	3	4	5	6	7	8
TSSV	228	183	171	166	160	155	151	147
TSN (Cuml)	228	411	582	748	908	1063	1214	1361

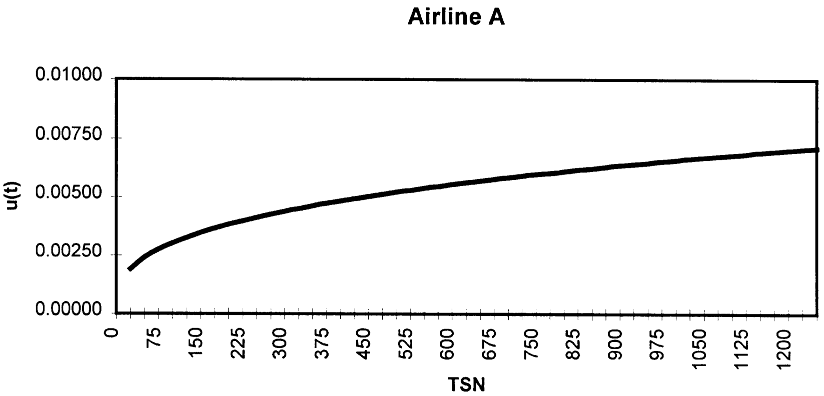

Fig. 6. Intensity function — mature engine.

flying hours resulting in a jet engine removal. The basic model design considers a number of factors as illustrated in Fig. 7.

The intensity function established for the early years, along with the mature expectations of the engine program were incorporated into the model. Fig. 8 illustrates the logic used in the model to determine the removal time (Event) of an engine while taking into consideration factors such as seasonal impact, spares influence, infant mortality and mandated removals (life limited hardware).

5.4. Seasonal effect

A goal of the model was to forecast the removals on a monthly basis; therefore the potential of seasonal effects was investigated. The outside ambient temperature has an impact on the engine's exhaust gas temperature (EGT). Typically, as the outside temperature increases, so does the EGT and vice versa. "High EGT" is often a cause for engine removals. For key airports, the Average Daily Maximum (ADM) were recorded on a monthly basis within a typical year to track ambient temperatures.

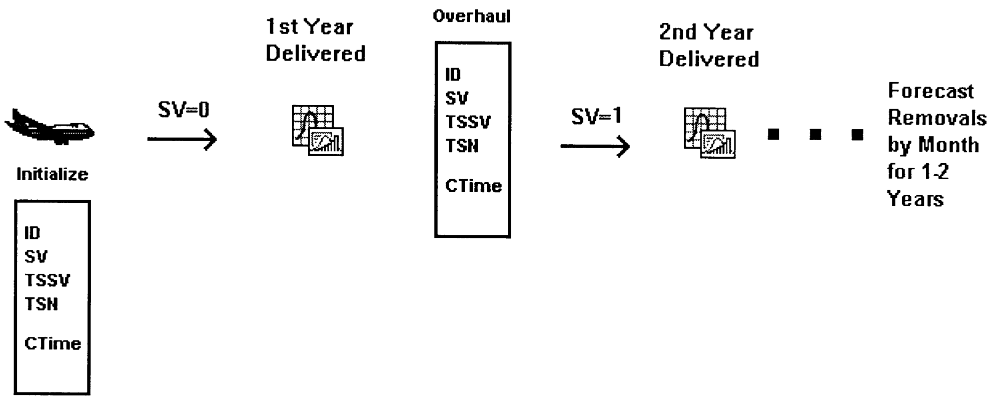

Fig. 7. Simulation design.

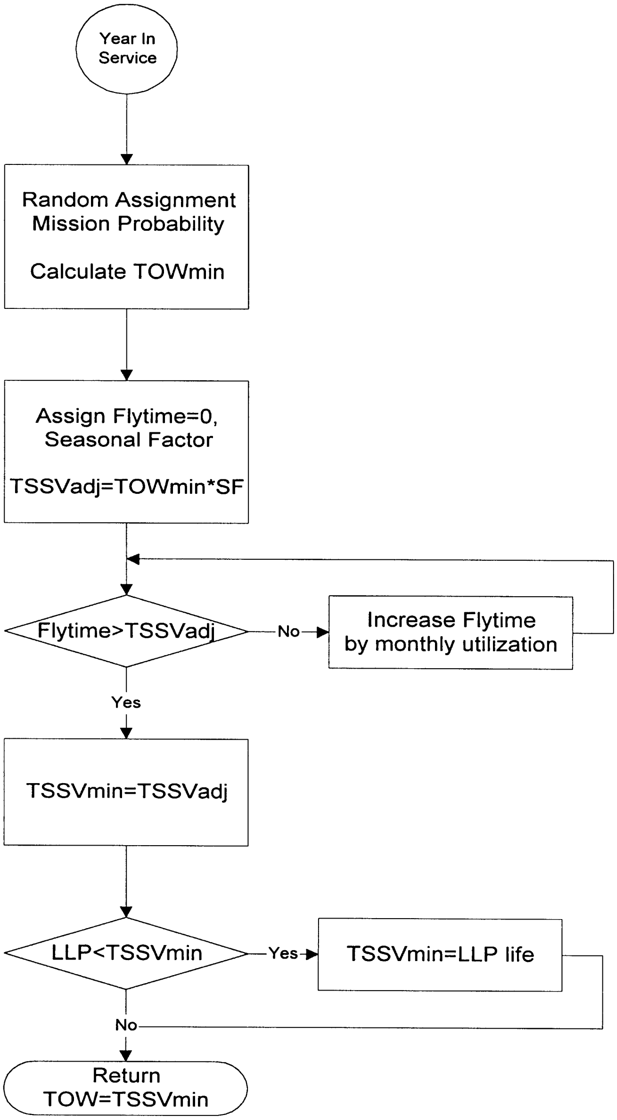

Fig. 8. Flowchart of model logic.

The seasonal effect was placed into the simulation model to reflect the change in temperature during different months of the year. The adjustment factor was determined for each month and applied to the TSSV of the engine flying. The seasonal impact was used to adjust the TSSV value calculated from the intensity function for a given jet engine. From a modeling aspect, the influence of ADM temperature was used in the simulations by calculating a ratio for each month and weighting this ratio with respect to TSSV. Through this method, the seasonality was considered as to how it influenced the life of a jet engine within the simulation phase.

Table 3
Forecast based on seasonal factors versus actual removals

Model	Airline A		Airline B	
	MSE	MAD	MSE	MAD
No seasonal effect	5.6	1.8	2.2[a]	1.1
1.25% Seasonal weight	5.3	1.7	2.4	1.2
2.50% Seasonal weight	5.3[a]	1.6	2.5	1.2

[a] Best fit based on historical data.

6. Measurement of fit and overall accuracy

Since a seasonal effect was considered, a number of simulations were completed to determine which seasonal factors "best fit" the actual removal events. To determine "best fit" the forecasted results were compared to actual removals over the same time period. Mean Squared Error (MSE) and Mean Absolute Deviation (MAD) were used as a measurement of fit. Table 3 lists their respective MSE and MAD values.

After determining which models seemed to fit the "best", the next step was to compare the model's forecast to actual results. In this case, an analysis was made for the "best fit" models by comparing their forecast to actual removals over an additional 27-month period. By chance, each airline had a different removal trend over this time period. Airline A's removal pattern flattened off from the prior months' rate of increase compared to Airline B's removal pattern which continued at a steady rate of increase.

To measure overall accuracy of the various models, the differences between the quantity of removals was calculated — Cumulative Absolute Deviation (CAD):

$$\mathrm{CAD} = \mathrm{CFR} - \mathrm{CAR} \tag{9}$$

where CFR = Cumulative Forecasted Removals and CAR = Cumulative Actual Removals.

In addition, another measurement that penalized a system for these continued deviations was also

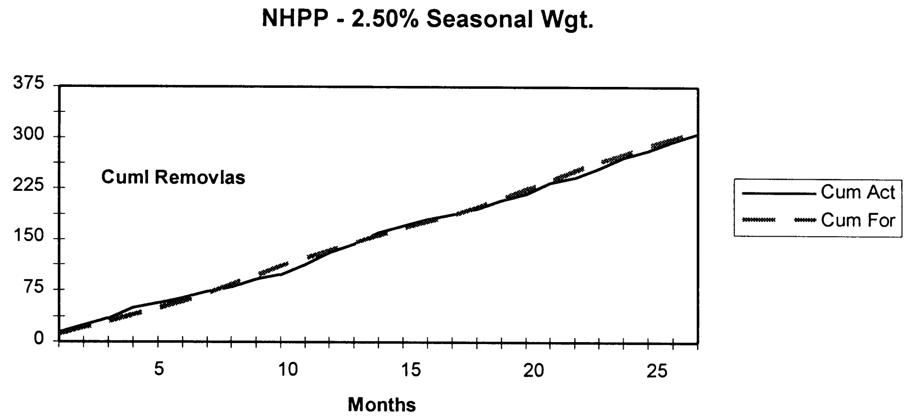

Fig. 9. Model fit to cumulative data.

Table 4
Model accuracy

Best model	Airline A		Airline B		Overall model accuracy (%)
	CAD	%	CAD	%	
NHPP	3 SV	1	34 SV	19	10

considered — Cumulative Mean Square Error (CMSE):

$$\mathrm{CSME} = \frac{\sum_{t=1}^{n} (\mathrm{CAR}_t - \mathrm{CFR}_t)^2}{n}. \tag{10}$$

Fig. 9 compares how one models forecast to actual removals for the 27-month period. During the next 27 months, Airline A had 292 shop visits while Airline B had 184 shop visits. The final result of the forecasted removal versus actual removals by each airline is shown in Table 4.

7. Conclusions

The Weibull process quite accurately predicted the SV outcome to within 1% for Airline A. However, its ability to predict the outcome of Airline B was less accurate. In the latter case, it seems that the significant number of mandated removals distorted the "counting process". In many situations, the mandated removals would result in an engine being prematurely removed due to a cycle-limited part versus removals due to engine deterioration or part failure. These events resulted in a TSSV that was underestimated, creating a higher than expected number of removals.

To obtain sufficient amount of data in jet engines would require five or more replacements and possibly take as long as 15 or more years. A key limitation of this methodology therefore is its sensitivity to the amount of sequential data required to calculate a usable intensity function. Since the average economical lifetime of an engine being about 20 years, the usefulness of these data is limited in the early years of a program. The utility function is clearly dependent on the level of knowledge of the individual involved in predicting TSSVs for subsequent removals. When the data are collected in the early stages of the engine program, and the knowledge base of the company is established, this could be an extremely effective forecasting tool. This is especially true when considerations are made for engine design. Derivatives for newer jet engine models will typically take on similar reliability characteristics.

This methodology, as currently applied for Airline B cannot account for all problems, such as the mandated removals or other interfering events that distort the "counting" process. Future research should investigate methodologies that could account for this distortion of the counting process. This method must be capable of estimating parametric values that could more accurately model the Weibull process. Unfortunately, this would limit an airline's ability to use this forecasting methodology if flying very short flight lengths otherwise the mandated removals become a very small percent of removals.

The Weibull process can be a very effective forecasting tool due to its ease of development and application. The Weibull process has considerable potential as a forecasting tool based on the results shown above. Its ability to forecast to within even 5% could save an airline substantially in the cost of spare engines that are typically needed for protection due to current forecasting techniques which have much higher forecasting errors. This, in itself, would be reason enough for its use.

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